647510 **0301207C1010** **Seat No :________**

B.Sc. Semester - 2 (*CBCS*) Examination
March/April - 2024 (As Per Syllabus Year-2016)
MATHEMATICS (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer all four. (04)

(1) Centre of the sphere $x^2+y^2+z^2-2x-4y-6z-11=0$ is _________.
(2) Radius of the sphere $x^2+y^2+z^2-2x-4y-6z-11=0$ is _________.
(3) Find the sphere for which (1,-1,1) and (-1, 1,1) are the extremities of diameter.
(4) Equation of cylinder whose axis is parallel to X-axis and radius r is _______.

Que.1(B) Answer any one out of two. (02)

(1) Find the centre and radius of the sphere $|\bar{r}|^2 + \bar{r}\,.\,(-8,6,-10) +1 = 0$
(2) Find equation of sphere passing through the circle $x^2+y^2+z^2=9$, $2x+3y+4z=5$ and the point (1,2,3) .

Que.1(C) Answer any one out of two. (03)

(1) Find the equation of the sphere passing through the points (0,0,0), (-a,b,c), (a,-b,c), (a,b,-c).
(2) Find the equation of cylinder whose generator is parallel to $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and passing through $x^2+xy+y^2=1$, z=0.

Que.1(D) Answer any one out of two. (05)

(1) Find centre and radius of the circle $x^2+y^2+z^2-x+z-2=0$, $x+2y-z=4$.
(2) Find the equation of the sphere passing through the points (4,-1,2), (0,-2,3), (1,5,-1), (2,0,1).

Que.2(A) Answer all four. (04)

(1) Find $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ if exist.
(2) $\lim\limits_{(x,y)\to(0,0)} \frac{tan(x+y)}{x+y}$ = _______.
(3) If f(x,y)= x tan(y/x) then find $f_y\,(4,\pi)$.
(4) If f(x,y)= log (x^2+y^2) then find $\left(\frac{\partial f}{\partial x}\right)_{(1,1)}$

Que.2(B) Answer any one out of two. (02)

(1) If f(x,y)= $\frac{x(x^2-y^2)}{(x^2+y^2)}$;$(x,y) \neq (0,0)$ and f(0,0)=0, then find $f_x\,(0,0)$
(2) If f(x,y)= x tan(y/x) then find $f_x\,(4,\pi)$.

Que.2(C) Answer any one out of two. (03)

(1) If w= $\frac{e^{x+y+z}}{e^x+e^y+e^z}$ then find the value of $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}$
(2) Find $\frac{dy}{dx}$ for the implicit function $\sin(xy)= e^{xy}+ x^2y$.

Que.2(D) Answer any one out of two. (05)

(1) If z (x+y)=(x^2+y^2), show that $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2 = 4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$.
(2) If f(x,y)=0 then obtain $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$.

Que.3(A) Answer all four. (04)

(1) Jacobian for $x=r\cos\theta$, $y=r\sin\theta$ is ______.
(2) If f(x,y)= x^2+2y^2-x then $f_x=0$ at x=_____.
(3) Find $f_{xy}\,(0,0)$ for f(x,y)= $e^x\cos y$.
(4) Find $f_{xx}\,(1,1)$ for f(x,y)= $x^2y+3y-2$.

Que.3(B) Answer any one out of two. (02)

(1) Find Jacobian for cylindrical coordinates.
(2) If u= 2x+y+z, v=x+2y+z, w= x+y+2z then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

Que.3(C) Answer any one out of two. (03)

(1) If u=x^2-2y, v=x+y+z, w= x-2y+3z then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

(2) If f(x,y) = x^3 +xy+y^3 then find the approximate value of f(1.01,2.98).

Que.3(D) Answer any one out of two. (05)

(1) If u= $\frac{x+y}{1-xy}$, v= $\tan^{-1} x + \tan^{-1} y$, then find $\frac{\partial(u,v)}{\partial(x,y)}$

(2) Expand log xy in power of x-1 and y-1.

Que.4(A) Answer all four. (04)

(1) Find A^2 for A= $\begin{bmatrix} -1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(2) If A= $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ then $|A|$ = ______.

(3) If A=$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$, B=$\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ then find AB.

(4) Give an example of upper triangular matrix.

Que.4(B) Answer any one out of two. (02)

(1) Show that there exist unique inverse matrix for any nonsingular matrix.

(2) Find the rank of matrix $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$

Que.4(C) Answer any one out of two. (03)

(1) If A= $\begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is orthogonal then find value of l,m,n.

(2) Find the rank of matrix $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

Que.4(D) Answer any one out of two. (05)

(1) In usual notation, prove that $(A+B)^T=A^T+B^T$ and $(AB)^T= B^T A^T$.

(2) Find rank of matrix $\begin{bmatrix} 3 & 1 & 4 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$ using echelon form .

Que.5(A) Answer all four. (04)

(1) Find eigen values of matrix A= $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

(2) Find characteristic equation of matrix A= $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$.

(3) Define eigen values.

(4) Define eigen vectors .

Que.5(B) Answer any one out of two. (02)

(1) Find characteristic equation of matrix A= $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(2) Find eigen values of matrix A= $\begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$.

Que.5(C) Answer any one out of two. (03)

(1) Find eigen values and eigen vectors of matrix A= $\begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$.

(2) Find eigen values and eigen vectors of matrix A= $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$.

Que.5(D) Answer any one out of two. (05)

(1) State and prove Cayley- Hamilton theorem.

(2) Verify Cayley- Hamilton theorem for $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

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